

17EC54

Fifth Semester B.E. Degree Examination, Jan./Feb. 2021 Information Theory and Coding

Time: 3 hrs .
Max. Marks: 100
Note: Answer any FIVE full questions, choosing ONE full question from each module.

## Module-1

1 a. Obtain an expression for average information content of long independent messages.
(05 Marks)
b. A block and white TV picture consists of 256 lines of picture information. Assume that each line consists of 526 picture elements and that each can have 255 brightness levels. Picture is repeated at the rate of 30 frames $/ \mathrm{sec}$. Calculate the average rate of information conveyed by TV picture.
(05 Marks)
c. For the Markov model shown in Fig.Q1(c). Find :
i) State probabilities
ii) State and source entropy
iii) $\mathrm{G}_{1}, \mathrm{G}_{2}$ and show that $\mathrm{G}_{1}>\mathrm{G}_{2}>\mathrm{H}$.


Fig.1(c)
(10 Marks)

## OR

2 a. A source emits one of four probable messages $M_{1}, M_{2}, M_{3}$ and $M_{4}$ with probabilities of $7 / 16,5 / 16,1 / 8$ and $1 / 8$ respectively. Find the entropy of the source. List all the elements of second order extension of this source. Hence show that $\mathrm{H}\left(\mathrm{s}^{2}\right)=2 \mathrm{H}(\mathrm{s})$.
b. Define the following :
i) Unit of information
ii) Entropy
iii) Self information
iv) Information rate.
(04 Marks)
c. For the Markoy model shown in Fig.Q2(c). Find :
i) State probabilities
ii) State entropy
iii) Source entropy
iv) Rate of information if $\mathrm{r}_{\mathrm{s}}=1 \mathrm{sym} / \mathrm{sec}$.


Fig.Q2(c)
(10 Marks)

## Module-2

3
a. Using Shannon's binary encoding algorithm, find all the codewords for the symbols, $\mathrm{P}=\{0.55,0.15,0.15,0.1,0.05\}$. Also find its efficiency and redundancy.
(10 Marks)
b. Consider the source, $\mathrm{S}=\{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}\}$ with $\mathrm{P}=\{0.1,0.15,0.25,0.35,0.08,0.07\}$. Find the codewords for the source using Shannon - Fano algorithm. Also find the source efficiency and redundancy.
c. Encode the following information using LZ algorithm : "THIS_IS_HIS_HIT".

## OR

4 a. An information source has a sequence of independent symbols with probabilities as follows : $\mathrm{S}=\{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}, \mathrm{G}, \mathrm{H}\}$ $P=\{0.4,0.25,0.12,0.08,0.05,0.05,0.03,0.02\}$.
Construct binary and ternary code using Huffman encoding procedure and find its efficiency redundancy.
( 10 Marks)
b. Explain prefix coding and Kraft - McMillan inequality with an example. Also draw the decision diagram for the prefix codes.
(05 Marks)
c. Consider a discrete memoryless source with $\mathrm{S}=\{\mathrm{X}, \mathrm{Y}, \mathrm{Z}\}$, probabilities $\mathrm{P}=\{0.5,0.3,0.2\}$. Find the code word for the message "YYZXZY" using arithmetic coding.
(05 Marks)

## Module-3

5 a. For the joint probability matrix given find:
i) $\mathrm{H}(\mathrm{X})$
ii) $\mathrm{H}(\mathrm{Y})$
iii) $\mathrm{H}(\mathrm{X}, \mathrm{Y})$
iv) $\mathrm{H}(\mathrm{Y} / \mathrm{X})$
v) $\mathrm{H}(\mathrm{X} / \mathrm{Y})$
vi) $I(X, Y)$.

$$
\mathrm{JPM}=\mathrm{P}(\mathrm{X}, \mathrm{Y})=\begin{aligned}
& \mathrm{x}_{1} \\
& \mathrm{x}_{2} \\
& \mathrm{x}_{3} \\
& \mathrm{x}_{4}
\end{aligned}\left[\begin{array}{ccccc}
0.05 & 0 & 0.20 & 0.05 \\
0 & 0.10 & 0.10 & 0 \\
0 & 0 & 0.20 & 0.10 \\
0.05 & 0.05 & 0 & 0.10
\end{array}\right]
$$


(10 Marks)
b. Prove that mutual information is always positive.
(05 Marks)
c. Find the channel capacity of the channel shown in Fig.Q5(c), by Muroga's method given $\mathrm{p}\left(\mathrm{x}_{1}\right)=0.6, \mathrm{p}\left(\mathrm{x}_{2}\right)=0.4$.


Fig.Q5(c)
(05 Marks)

## OR

6 a. Obtain an expression for the channel capacity of binary symmetric channel.
(05 Marks)
b. For a channel matrix $p(Y / X)=\left[\begin{array}{ll}3 / 4 & 1 / 4 \\ 1 / 4 & 3 / 4\end{array}\right]$, given $p\left(x_{1}\right)=2 / 3, p\left(x_{2}\right)=1 / 3, r_{s}=1000$ sym/sec. Find $\mathrm{H}(\mathrm{X}), \mathrm{H}(\mathrm{Y}), \mathrm{H}(\mathrm{X}, \mathrm{Y}), \mathrm{H}(\mathrm{Y} / \mathrm{X}), \mathrm{H}(\mathrm{X} / \mathrm{Y}), \mathrm{I}(\mathrm{X}, \mathrm{Y})$ and channel capacity, information rate.
(10 Marks)
c. Define mutual information and prove that $\mathrm{H}(\mathrm{X} / \mathrm{Y})=\mathrm{P} \cdot \mathrm{H}(\mathrm{X})$ for a binary erasure channel.
(05 Marks)

## Module-4

7 a. For a systematic $(7,4)$ linear block code, parity matrix is given by,
i) Find all the possible valid code words
ii) Draw the encoding and syndrome calculation circuit.
iii) A single error has occurred in each of the following code words given,
iii) A single error has occurred in each of the following code words given,

$$
\mathrm{P}=\left[\begin{array}{lll}
1 & 1 & 1 \\
1 & 1 & 0 \\
1 & 0 & 1 \\
0 & 1 & 1
\end{array}\right]
$$

(10 Marks)
b. The generator polynomial of a $(7,4)$ cyclic code is $g(x)=1+x+x^{3}$, find the codewords for message vectors (1010), (1110), (1100) and (1111) using systematic and non-systematic form.
(10 Marks)

## OR

8 a. A $(6,3)$ linear block code has the following check bits $C_{4}=d_{1}+d_{2}, C_{5}=d_{1}+d_{3}$ and $\mathrm{C}_{6}=\mathrm{d}_{2}+\mathrm{d}_{3}$.
i) Write the G and H matrices
ii) Draw the encoding and syndrome calculation circuits.
iii) Construct the standard array and through example illustrate decoding operation.(10 Marks)
b. Consider $(15,5)$ linear cyclic code with generator polynomial,
$g(x)=1+x+x^{2}+x^{4}+x^{5}+x^{8} x^{10}$
i) Draw the encoder and syndrome circuit
ii) Find the code vector for the message polynomial $d(x)=1+x^{2}+x^{4}$ by listing the states of the shift registers.
iii) Is $\mathrm{V}(\mathrm{x})=1+\mathrm{x}^{4}+\mathrm{x}^{6}+\mathrm{x}^{8}+\mathrm{x}^{14}$ a code polynomial? If not find the syndrome. ( $\mathbf{1 0} \mathrm{Marks}$ )

## Module-5

9 a. Consider $\mathrm{a}(3,1,2)$ convolution code with impulse responses $\mathrm{g}^{(1)}=(110), \mathrm{g}^{(2)}=(101)$, $\mathrm{g}^{(3)}=(111)$.
i) Draw the encoder diagram
ii) Find the generator matrix
iii) Find the code vector for the information sequence (11101) using time domain and transform domain approach.
(10 Marks)
b. Write short notes on :
i) BCH code
ii) Golay codes.

## OR

10 a. Consider the convolutional encoder shown in Fig.Q10(a).
i) Draw the state table, state transition table and state diagram
ii) Using the code tree, find the encoded sequence for the message vector (10111)
iii) Verify the output sequence so obtained using transform domain approach.


Fig.Q10(a)
(10 Marks)
b. Explain viterbi decoding algorithm with an example.

